International Journal of Theoretical Physics, Vol. 8, No. 2 (1973), pp. 141–146

The Exclusive Interaction

WILLIAM A. BARKER

Physics Department, University of Santa Clara, Santa Clara, California

Received: 10 *July* 1972

Abstract

It is proposed that there exists a fifth basic interaction from which the Pauli exclusion principle can be deduced. Such an interaction would affect only particles of half integral spin, whose total number is less than the number of particles which participate directly in the strong interaction. This is the reason for choosing the name exclusive interaction, which, like the strong interaction, would be expected to obey a relatively large number of conservation laws and symmetry principles. The tentative mathematical expression for this interaction

$$V(r,\sigma) = \frac{g}{r}\cos(cr)\exp(-r/R)\left[\frac{1+\vec{\sigma}_1.\vec{\sigma}_2}{2}\right]$$

is suggested by a discussion of fermion–fermion scattering in the Born approximation. It is found that a potential energy of this form may be used instead of antisymmetrizing the wave function in the manner required by the Pauli principle. An investigation of the binding of helium-like atoms is expected to lead to a determination of the relative strength $g/\hbar c$ and the range R of this interaction.

1. *Introduction*

The essential idea of this paper is that there exists a basic interaction which, when properly formulated, accounts for the Pauli exclusion principle (Pauli, 1925). This postulated interaction bears the same relationship to Pauli's empirical rule as the gravitational interaction does to Kepler's laws of motion. This is not a new idea. The need for a fundamental theory of the interaction of two equivalent particles of half integral spin is mentioned by Condon & Shortley (1951) in their well-known book. This possibility is explored philosophically by Margenau (1950) and has been discussed briefly at meetings of the American Physical Society (Barker, 1969; Barker & Sears, 1970).

In any treatment of a possible new interaction, it is instructive to examine the general features which characterize the four recognized basic interactions. We find that strong, electromagnetic, weak and gravitational forces are described† in terms of relative *strength*, *range* and '*charge*',

† See for example, Chew, G. *et al.* (1964).

particles directly affected and conservation laws and symmetry principles obeyed. There is a very interesting correlation† between the strength of an interaction, the number of particles directly involved and the number of conservation laws violated. This connection provides a clue in the description of the exclusive interaction.

The strong interaction affects the smallest number of particles and obeys the largest number of conservation laws and symmetry principles. As the strength of the interactions diminish more particles participate, but at the expense of violating more conservation laws and with longer characteristic reaction times.

The Pauli exclusion principle applies only to particles whose spin is $\hbar/2$, $3\hbar/2$, etc. Among the metastable particles (Rosenféld *et al.*, 1965), this includes, if we count particles and antiparticles, the four neutrinos, the electron and positron, the positive and negative muon, the proton and antiproton, the neutron and anti-neutron, the two lambdas, the six sigmas, the four Xis and the two omegas, a total of twenty-six. The strong interaction directly involves the eighteen baryons mentioned and nine different mesons, for a total of twenty-seven. If we include in this comparison the large number of particles which decay very rapidly by the strong interaction, we find that the total number of known fermions is considerably smaller than the total number of strongly interacting particles. It is for this reason that the name exclusive interaction has been chosen. Further, this comparison suggests that the exclusive interaction will probably have a strength comparable to the strong or at least the electromagnetic interaction and will probably obey a relatively large number of conservation laws and symmetry principles.

2. *Scattering in the Born Approximations*

In order to get a specific idea regarding the quantitative form of the exclusive interaction, we consider the scattering of two fermions in the Born approximation. The conventional quantum mechanical treatment of this problem requires that the wave function which describes the two particles be antisymmetrized to be consistent with the Pauli principle. Is it possible to add an interaction potential energy to the wave equation which leads to the same scattering amplitude without antisymmetrizing the wave function?

2.1. *Fermion–Fermion Scattering. Outline of Conventional Treatment Using Antisymmetrization of the Wave Function*

The scattering amplitude $f(\theta)$ in the center of a mass coordinate system can be found from the asymptotic form of the solution of the wave equation.

$$-\frac{\hbar^2}{2\mu}\nabla^2 u+\frac{e^2}{2}u=Eu \tag{2.1.1}$$

† This unexplained correlation is discussed for example, by Ford, Kenneth (1968).

Here, r is the radial distance between two particles of the same mass m and charge e. The reduced mass for the system $\mu = m/2$. The wave function

$$u \xrightarrow[r\to\infty]{} \exp(ikz) + r^{-1}f(\theta)\exp(ikr) \tag{2.1.2}$$

where the propagation vector $\mathbf{k}$ has the magnitude $k = \mu v/\hbar$ and is directed along the polar axis. The relative speed of the two particles is v. In the Born approximation

$$f(\theta) = -\frac{2\mu}{\hbar^2 K}\int_0^\infty rV(r)\sin(Kr)\,dr \tag{2.1.3}$$

where $\hbar K = 2k\hbar\sin(\theta/2)$ is the magnitude of the momentum transferred from a particle of mass μ to the scattering potential during the collision. We consider two cases: scattering by a screened Coulomb potential

$$V(r) = \frac{e^2}{r}\exp(-r/a) \tag{2.1.4}$$

and by a potential barrier

$$\begin{aligned} V(r) &= V_0 \qquad (0 \leqslant r \leqslant a) \\ V(r) &= 0 \qquad (r > a) \end{aligned} \tag{2.1.5}$$

Explicit evaluation of equation (2.1.3) yields

$$f(\theta) = -\frac{2\mu e^2}{\hbar^2(K^2 + a^{-2})} \tag{2.1.4a}$$

and

$$f(\theta) = -\frac{2\mu V_0}{\hbar^2 K^3}(\sin Ka - Ka\cos Ka) \tag{2.1.5a}$$

These results are valid in the approximation $ka \gg 1$.†
The scattering amplitude expression, for $Ka \gg 1$, reduces to

$$f(\theta) = -\frac{2\mu e^2}{\hbar^2 K^2} \tag{2.1.4b}$$

and

$$f(\theta) = -\frac{2\mu V_0 a}{\hbar^2 K^2} \tag{2.1.5b}$$

The results for the two cases are seen to be the same if the range a is chosen the same and $V_0 = e^2/a$. In the case of non-identical particles of charge $|e|$, this leads to the classical Rutherford result for the differential scattering cross section.

$$\sigma(\theta) = |f(\theta)|^2 = \frac{e^4}{4\mu^2 v^4}\operatorname{cosec}^4\frac{\theta}{2} \tag{2.1.6}$$

† See, for example, Schiff, L. I. (1955).

However, for identical particles of spin-$\frac{1}{2}\hbar$, the Pauli principle (Schiff, 1955, p. 228) requires that the spatial part of the asymptotic wave function be symmetric (antisymmetric) if the spin part is antisymmetric (symmetric).

$$u \xrightarrow[r\to\infty]{} \exp(ikz) \pm \exp(-ikz) + [f(\theta) \pm f(\pi-\theta)]\, r^{-1} \exp(ikr) \quad (2.1.7)$$

This equation fulfills the formal requirement that the total wave function be antisymmetric under an exchange of the two particles. The plus sign is used in equation (2.1.7) when the particles have antiparallel spins and are described by an antisymmetric singlet spin wave function. The minus sign is used, on the other hand, when the particles have parallel spins and are described by a symmetric triplet spin wave function. We note that for Coulomb scattering

$$f(\pi-\theta) = -\frac{2\mu e^2}{\hbar^2(K'^2 + a^{-2})} \approx -\frac{\mu e^2}{2\hbar^2 k^2 \cos^2(\theta/2)} \quad (2.1.8a)$$

and for square barrier scattering

$$f(\pi-\theta) = -\frac{2\mu V_0}{\hbar^2 K'^3}(\sin K' a - K' a \cos K' a) \approx -\frac{\mu V_0 a}{2\hbar^2 k^2 \cos^2(\theta/2)} \quad (2.1.8b)$$

where $K' = 2k\cos(\theta/2)$.

2.2. *Fermion–Fermion Scattering. Outline of New Treatment in which an Interaction Potential Energy is Used Instead of Antisymmetrizing the Wave Function*

We now inquire as to the form of an interaction potential energy which could be added to equation (2.1.1) which would lead to both the positive and negative values of the scattering amplitudes expressed in equations (2.1.8). We see, from equation (2.1.3) that this is equivalent to finding a $V(r,\sigma)$ such that

$$\int_0^\infty rV(r,\sigma)\sin(Kr)\,dr = \pm\frac{Ke^2}{K^2-4k^2} \quad \text{or} \quad \pm\frac{KV_0 a}{K^2-4k^2} \quad (2.2.1)$$

The algebraic signs in equation (2.2.1) may be obtained by letting the spatial part of the potential energy be multiplied by the spin exchange operator.†

$$V(r,\sigma) = \left[\frac{1+\sigma_1.\vec{\sigma}_2}{2}\right] V(r) \quad (2.2.2)$$

Hence for parallel spins $V = +V(r)$ and for antiparallel spins $V = -V(r)$. The spatial part of the potential energy may be found by using the theory of Fourier transforms (Morse & Feshback, 1953). Let

† See for example, Bohm, David (1951).

$$F(K)=\left(\frac{2}{\pi}\right)^{1/2}\int_0^\infty \sin(Kr)\,rV(r)\,dr=\left(\frac{2}{\pi}\right)^{1/2}\frac{Ke^2}{K^2-4k^2} \quad \text{or}$$

$$\left(\frac{2}{\pi}\right)^{1/2}\frac{KV_0a}{K^2-4k^2} \tag{2.2.3}$$

Then

$$rV(r)=\left(\frac{2}{\pi}\right)^{1/2}\int_0^\infty \sin(Kr)F(K)\,dK \tag{2.2.4}$$

The integral over K may be performed by making the change of variables $x=Kr$ and letting $\sigma=kr$, and noting that the Cauchy principal value (Arfken, 1970) of the improper integral

$$\int_0^\infty \frac{x\sin x}{x^2-\sigma^2}\,dx=\pi\cos\sigma \tag{2.2.5}$$

The final expression for $V(r,\sigma)$ is then

$$V(r,\sigma)=\left[\frac{1+\vec{\sigma}_1.\vec{\sigma}_2}{2}\right]\cos(2kr)\frac{e^2}{r} \tag{2.2.6a}$$

or

$$V(r,\sigma)=\left[\frac{1+\vec{\sigma}_1.\vec{\sigma}_2}{2}\right]\cos(2kr)\frac{V_0a}{r} \tag{2.2.6b}$$

That equations (2.2.6) give the desired results may be confirmed by substituting $V(r,\sigma)$ directly into equation (2.1.3) and using the definite integral

$$\lim_{d\to 0}\int_0^\infty \exp(-dr)\sin br\cos cr\,dr=\lim_{d\to 0}\frac{b(d^2+b^2-c^2)}{[d^2+(b-c)^2]\,[d^2+(b+c)^2]} \tag{2.2.7}$$

with $b=K$ and $c=2k$.

The tentative mathematical form for the exclusive interactions, adaptable to fermions with or without electrical charge, is suggested by equations (2.2.6). Hence it is proposed that

$$V(r,\sigma)=\frac{g}{r}\cos(cr)\exp(-r/R)\left[\frac{1+\vec{\sigma}_1.\vec{\sigma}_2}{2}\right] \tag{2.2.8}$$

be used to investigate the binding energy of helium and helium-like atoms. This study should lead to estimates for the relative strength $g/\hbar c$ and range R of the exclusive interaction. The quantity c which appears in the argument of the cosine function will be treated as a parameter to be determined in each case by a variational approach similar to the methods first used by Hylleraas (1930).

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